

Base Pressure Behaviour in a Suddenly Expanded Duct at Supersonic Mach Number Regimes using Taguchi Design of Experiments

Jaimon Dennis QUADROS

*Department of Mechanical Engineering, Birla Institute of Technology
Off-shore Campus, Ras Al Khaimah-41222, UAE
e-mail: jaimonq@gmail.com*

Sher Afghan KHAN

*Department of Mechanical Engineering, International Islamic University Malaysia
Kuala Lumpur-50728, Malaysia*

A. J. ANTONY

*Department of Mechanical Engineering, Bearys Institute of Technology
Mangalore-574153, India*

Received (2 February 2018)

Revised (10 September 2018)

Accepted (20 November 2018)

Experimental investigations are carried out to study the control of base pressure without and with the use of micro-jets through suddenly expanded axi-symmetric passage in the supersonic regime. Four micro jets having an orifice diameter of 1mm were located at 90° intervals. In the base area, active controls jets have been placed on a pitch of a circle diameter that is 1.3 times the exit diameter of the nozzle. The jets were dispensed abruptly into the axi-symmetric tube maintained at a cross-sectional area of 4.84 times the exit nozzle area. The variation of base pressure as a function of flow control parameters namely Mach number, nozzle pressure ratio (NPR) and length to diameter) ratio (L/D) are evaluated experimentally. This study also assesses the impact of flow control variables on base pressure for two cases viz. with control and without control respectively. An L_9 orthogonal array of Taguchi and the analysis of variance were employed to investigate the percentage of contribution of these parameters and their interactions affecting the base pressure. The correlations between the various factors affecting the base pressure were obtained by using multiple linear regression equations. Confirmation tests were conducted in order to test the developed linear regression equations for their practical significance. Both the regression models were found to be significant and reliable with a percentage deviation lying in the range of –6.12% to 10.26% for base pressure without control and –13.92% to 6.58% for base pressure with control. Analysis of variance was also performed in order to determine the statistical significance of each parameter on the total variability of base pressure. The study concluded that Mach number is the most influential parameter affecting base pressure followed by NPR and L/D .

Keywords: micro-jets, Mach number, nozzle pressure ratio (NPR), length to diameter ratio, Taguchi

1. Introduction

The flow separation is said to encounter at the base of modern aerodynamic vehicles like missiles, rockets and projectiles. This separation of flow edges towards the establishment of low-speed recirculation zone very near to its base. Thus the pressure pertaining to this region is relatively lesser than the free stream atmospheric pressure. Such pressure difference can cause base drag which in turn tends to cover two-thirds of the total drag owing to a body undergoing revolutions. There are various techniques like burning of base and bleed of base which is used to lessen the base drag. However not very many studies have been conducted using active control. Suddenly expanded flows in supersonic Mach regimes have found extensive variety of utilizations. One such application includes shroud configuration in a form of parallel diffuser which is supersonic in nature. Similarly, the same flow behavior is observed in the combustion engine which includes a plane involving hot gasses moving through the exhaust valve. Stream field of axi-symmetric expansion is a complex episode and is described by the stream which is (i) separated; (ii) recirculated and (iii) reattached. Such a stream field is separated by a shear layer framing two principle zones one being the fundamental flow zone and the other being recirculation zone. The unity of striking of partitioning streamline is called as the reattachment point.

The fundamental features of the abruptly expanded flow field are shown in Fig. 1. The inception of the concept of expanded flows was formulated by [1] who studied the boundary layer effect on sonic flow experimentally. It was observed that, the pressure at the extended corner was principally reliant on boundary layer type and thickness. Here the boundary limit was considered to be the cause for fluid flow across the corner. Anderson et al. (computed the base pressure and noise established by the abrupt expansion of air in a cylindrical duct [2]. The appended flow consisted of base pressure possessing the most minimal value primarily reliant on the area ratio. Thus noise throughout was observed to be at minimum at a jet pressure comparatively equal to the one necessitated to attain minimal base pressure. Drag diminishment through axi-symmetric passage for Mach 2.0 was studied by Viswanath et al. [3]. Base cavities and ventilated cavities were examined in the devices. The results showed compelling base drag reduction that was offered by ventilated cavities. It was found that a 50% increase in base pressure and 5% base reductions was observed at supersonic Mach number. Badrinarayanan examined the base flows at supersonic speeds experimentally [4]. The measurements were done in the wake flow trailing the blunt based 2-D and 3-D bodies at $M = 2$. The results demonstrate the behaviour of separated flows and also point out the significance of flow reversal.

The development of air injection at the base recognizes that the base pressure increases significantly with air injection. Khan et al. carried out experiments in order to study the response of micro jets influencing over, under and correct expansion to control the base pressure in suddenly expanded axisymmetric ducts [5, 6, 7, 8 and 9]. The maximum increase in base pressure was 152 percent for Mach of 2.58. It was concluded that the micro jets do not have an adverse effect on the wall

pressure distribution. Furthermore, it was also concluded that the nozzle pressure ratio has a definitive role to play in controlling the variation in the base pressure for both cases i.e. with and without control. Suddenly expanded flows without and with control can be an interesting study owing to many applications. One of such applications include the space programme wherein behavior of base pressure is of prime importance to devise a mechanism to control base pressure in order to facilitate either its increase or decrease. This mechanism can eventually be used for high end applications such as minimizing base pressure in the ignition chamber for augmenting the blending; increase the base pressure if there should be an occurrence in rockets and missiles to bring about decrease in the base drag. Baig et al. conducted experimental investigations for manipulating base pressure through a suddenly expanded passage [10]. For this purpose, micro-jets were employed as active controllers for controlling base pressure. The Mach numbers employed were 1.87, 2.2 and 2.58. The area ratio was 2.56 and L/D from 10 to 1 were implemented for experimentation. The experiments were operated at NPRs ranging from 3 to 11 in steps of two. The study observed an increase in base pressure as high as 65 percent for certain parametric combinations.

From the literature survey, it is clear that there are no base pressures studies have been conducted as per Taguchi design of experiments. Thus the present study is novel and hasn't been conducted elsewhere. It attempts to conduct experiments as per Taguchi design and also enable the influence and significance of various parameters and their interactions on base pressure. The study also investigates the control of suddenly expanded flow with active control in the form of active and passive control. Furthermore, correlations between the various factors affecting the base pressure were obtained by using multiple linear regression equations. Analysis of variance (ANOVA) has also been employed in order to investigate the percentage of contribution of these parameters and their interactions affecting the base pressure.

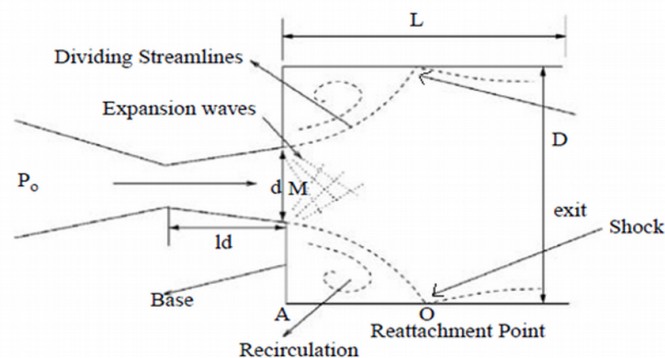


Figure 1 Suddenly expanded flow field

2. Mathematical Formulation

Nozzles come up in a vast range of applications. Obvious ones are the thrust nozzles of rocket and jet engines. Converging-diverging ducts also come up in aircraft engine inlets, wind tunnels and in all sorts of piping systems designed to control gas flow. The flows associated with volcanic and geyser eruptions are influenced by converging-diverging nozzle geometries that arise naturally in geological formations. From area-averaged equations of motion [11], by neglecting the shear stresses and heat fluxes, the governing equations together with the perfect gas law are given by

$$d(\rho UA) = 0 \quad (1)$$

$$dP + (\rho U dU) = 0 \quad (2)$$

$$C_p dT + (U dU) = 0 \quad (3)$$

$$P = \rho RT \quad (4)$$

Now, introducing Mach number

$$U^2 = \gamma RT M^2 \quad (5)$$

Equations (1–4) can be expressed in fractional differential form as

$$\frac{d\rho}{\rho} + \frac{dU^2}{2U^2} + \frac{dA}{A} = 0 \quad (6)$$

$$\frac{dP}{P} + \frac{\gamma M^2}{2} \frac{dU^2}{U^2} = 0 \quad (7)$$

$$\frac{dT}{T} + \frac{(\gamma - 1)M^2}{2} \frac{dU^2}{U^2} = 0 \quad (8)$$

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T} \quad (9)$$

Equation (5) can be expressed in fractional differential form as

$$\frac{dU^2}{U^2} = \frac{dM^2}{M^2} + \frac{dT}{T} \quad (10)$$

By using the equations for mass, momentum and energy to replace the terms in the equation of state, we get

$$-\frac{\gamma M^2}{2} \frac{dU^2}{U^2} = -\frac{dU^2}{U^2} - \frac{dA}{A} - \frac{(\gamma - 1)M^2}{2} \frac{dU^2}{U^2} \quad (11)$$

Solving for $\frac{dU^2}{U^2}$, we get

$$\frac{dU^2}{U^2} = \left(\frac{2}{M^2 - 1} \right) \frac{dA}{A} \quad (12)$$

Equation (12) shows the effect of stream wise area change on the speed of the flow. Using Eq. (12) to replace $\frac{dU^2}{U^2}$ in each of the relations in equations (6–8), we get

$$\frac{d\rho}{\rho} = \left(\frac{2}{M^2 - 1} \right) \frac{dA}{A} \quad (13)$$

$$\frac{dP}{P} = \left(\frac{\gamma M^2}{M^2 - 1} \right) \frac{dA}{A} \quad (14)$$

$$\frac{dT}{T} = \left(\frac{(\gamma - 1)M^2}{2} \right) \frac{dU^2}{U^2} \frac{dA}{A} \quad (15)$$

Equations (13–15) describe the effects of area change on the thermodynamic state of the flow. Now use equation (12) in temperature Eq. (10). We get

$$\begin{aligned} & \ln \left(\frac{A^*}{A} \right) = \\ & \ln \left(\left(\frac{\gamma + 1}{2} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \right) - \left\{ -\ln(M) + \ln \left(\left(1 + \left(\frac{\gamma-1}{2} \right) M^2 \right)^{\frac{\gamma+1}{2(\gamma-1)}} \right) \right\} \ln \left(\frac{A^*}{A} \right) = \\ & \ln \left(\left(\frac{\gamma + 1}{2} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \right) - \left\{ -\ln(M) + \ln \left(\left(1 + \left(\frac{\gamma-1}{2} \right) M^2 \right)^{\frac{\gamma+1}{2(\gamma-1)}} \right) \right\} \end{aligned} \quad (16)$$

Rearranging Eq. (16), the effect of area change on the Mach number is

$$\begin{aligned} \frac{dA}{A} &= \left(\frac{M^2 - 1}{2 \left\{ 1 + \left(\frac{\gamma-1}{2} \right) M^2 \right\}} \right) \frac{dM^2}{M^2} \frac{dA}{A} = \\ & \left(\frac{M^2 - 1}{2 \left\{ 1 + \left(\frac{\gamma-1}{2} \right) M^2 \right\}} \right) \frac{dM^2}{M^2} \end{aligned} \quad (17)$$

Integrate Eq. (17) from an initial Mach number M to one Eq. (18)

$$\begin{aligned} & \int_{M^2}^1 \left(\frac{M^2 - 1}{2 \left\{ 1 + \left(\frac{\gamma-1}{2} \right) M^2 \right\}} \right) \frac{dM^2}{M^2} = \\ & \int_A^{A^*} \frac{dA}{A} \int_{M^2}^1 \left(\frac{M^2 - 1}{2 \left\{ 1 + \left(\frac{\gamma-1}{2} \right) M^2 \right\}} \right) \frac{dM^2}{M^2} = \int_A^{A^*} \frac{dA}{A} \end{aligned} \quad (18)$$

We get Eq. (19)

$$\begin{aligned} & \ln \left(\frac{A^*}{A} \right) = \\ & \ln \left(\left(\frac{\gamma + 1}{2} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \right) - \left\{ -\ln(M) + \ln \left(\left(1 + \left(\frac{\gamma-1}{2} \right) M^2 \right)^{\frac{\gamma+1}{2(\gamma-1)}} \right) \right\} \ln \left(\frac{A^*}{A} \right) = \\ & \ln \left(\left(\frac{\gamma + 1}{2} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \right) - \left\{ -\ln(M) + \ln \left(\left(1 + \left(\frac{\gamma-1}{2} \right) M^2 \right)^{\frac{\gamma+1}{2(\gamma-1)}} \right) \right\} \end{aligned} \quad (19)$$

Evaluating Eq. (19) at the limits, we get the final equation as

$$\begin{aligned} \frac{A^*}{A} &= \left\{ \left[\frac{\gamma + 1}{2} \right]^{\frac{\gamma+1}{2(\gamma-1)}} \frac{M}{\left(1 + \left(\frac{\gamma-1}{2} \right) M^2 \right)^{\frac{\gamma+1}{2(\gamma-1)}}} \right\} \frac{A^*}{A} = \\ & \left\{ \left[\frac{\gamma + 1}{2} \right]^{\frac{\gamma+1}{2(\gamma-1)}} \frac{M}{\left(1 + \left(\frac{\gamma-1}{2} \right) M^2 \right)^{\frac{\gamma+1}{2(\gamma-1)}}} \right\} \end{aligned} \quad (20)$$

In the Eq. (20), we referenced the integration process to $M = 1$. The area A^* is a reference area at some point in the channel where $M = 1$ although such a point need not actually be present in a given problem. The area Mach number function is given by Eq. (20).

3. Experimental Setup

The setup is comprises of pipelines, control pressure valves and settling chamber. The dry air which is compressed at high pressure is admitted into the settling chamber via a pressure regulatory valve before its diffusion take place into the experimental models. The stagnation pressure level within the control chamber is restrained by a controlling valve. The dry air which is cultivated at a predetermined pressure is augmented through the nozzle; in this manner succeeding into the abruptly expanded pipe. The air along these lines leaving the pipe is scattered into the surrounding atmosphere.

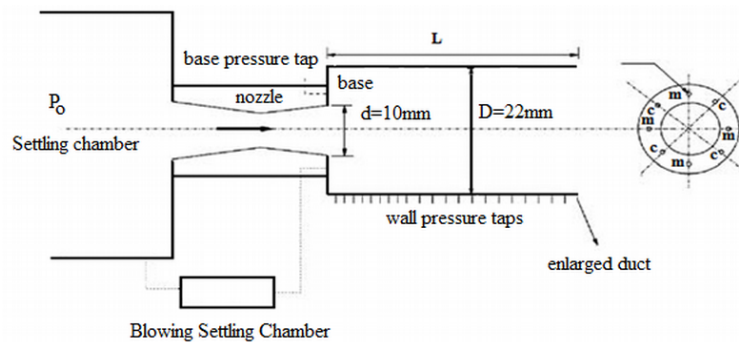


Figure 2 Experimental setup

Figure 2 demonstrates the test setup that has been utilized to lead the present study. At the outer rim, there are eight nozzle openings set apart of 1mm diameter each, four have been used for blowing and the remaining four for base pressure measurement P_b . Base pressure was controlled by agitating air through control holes (c), using the settling chamber pressure by engaging tube that connects the control chamber with the settling chamber. The experiments have been carried out for Mach numbers 2.0, 2.5 and 3.0. Dynamic controls as a micro-jet form have been utilized. For each Mach numbers L/Ds of 3, 5 and 8 have been employed. The NPR's employed are 3, 5 and 7 respectively. NPR is defined as the ratio of stagnation pressure P_0 , i.e. pressure in the settling chamber to ambient atmospheric pressure P_a . NPR also happens to be condition for micro jet better known as the micro jet blow pressure ratio. Here the pressure from the settling chamber is used in the form of micro jet pressure through a blowing settling chamber by blowing through the control holes that are fixed at the nozzle exit as shown in the experimental set up.

Convergent-divergent nozzles of 10mm common exit diameters and throat diameter of 7.7, 6.15, and 4.86 mm, correspond to design Mach numbers of 2.0, 2.5, and 3.0 were fabricated. The calibration of these nozzles delivered flows of Mach

2.0, 2.5, and 3.0, respectively which were same as the design Mach numbers. The enlarged duct was a consistent metal tube made of brass having a diameter of 22mm which relates to an area proportion of 4.84. The broadened duct and regions at the base were infiltrated with pressure taps as appeared in the test setup.

Measurements concerning base pressure are been taken along the base and wall of the duct length for two cases (i) without operating the microjets i.e. without control and (ii) microjets on i.e. with control. The pressure transducer of the make PSI System 2000 was used for measuring pressure at the base and as well as stagnation pressure. It has 16 channels and the pressure range is 0–300psi. It averages 250 samples per second and displays the reading. Mercury manometer was used for measurement of duct wall pressure distribution. User friendly software has been used to acquire data from all the simultaneous from all 16 channels simultaneously and displays it on the computer screen. The transducer is operated in temperatures ranging from -20 to $+600$ and 95% humidity. The transducer had a measurement resolution of ± 0.003 and the readings were accurate upto ± 1 percent. Mercury manometer was used for the measurement of wall pressure.

3.1. Design of the Nozzle

The nozzle design for a Mach number of 2.0, 2.5 and 3.0 involves a certain set of parameters that are standardized for its design. The parameters involved are:

1. Exit diameter of the nozzle is fixed at 10mm.
2. Throat diameter determined is from the by Eq. (20). For instance, for Mach number 3 (ratio of exit area to throat area, i.e. $A_e/A^* = 4.235$).
3. The angle between the exit diameter and throat of the nozzle is maintained at an angle of approximately 5° to 8° .
4. The angle between the inlet diameter and throat is maintained at an angle of approximately 18° to 30° . Based on the standard parameters mentioned, the nozzles are designed for the specified Mach numbers 2.0, 2.5 and 3.0 with detailed dimensions. The nozzle design for Mach 3.0 has been presented in Fig. 3.

4. Plan of Experiments

The Taguchi procedure for outline of investigations has been utilized to arrange the analyses comprising of three variable and three levels. The steps for implementation of Taguchi are shown in Fig. 4. A typical L_9 orthogonal array including 9 rows and 4 columns has been used and shown in Tab. 1.

The flow parameters regarded in the current investigation are Mach number, NPR and L/D . The examination comprises of nine tests comparing to the quantity of rows and columns that have been allotted to the flow parameters. The primary column is allocated to Mach number, second to NPR and the third to L/D . The parameters with their levels are as shown in Tab. 2. The obtained test results for experiments conducted as per L_9 orthogonal array have been shown in Tab. 3.

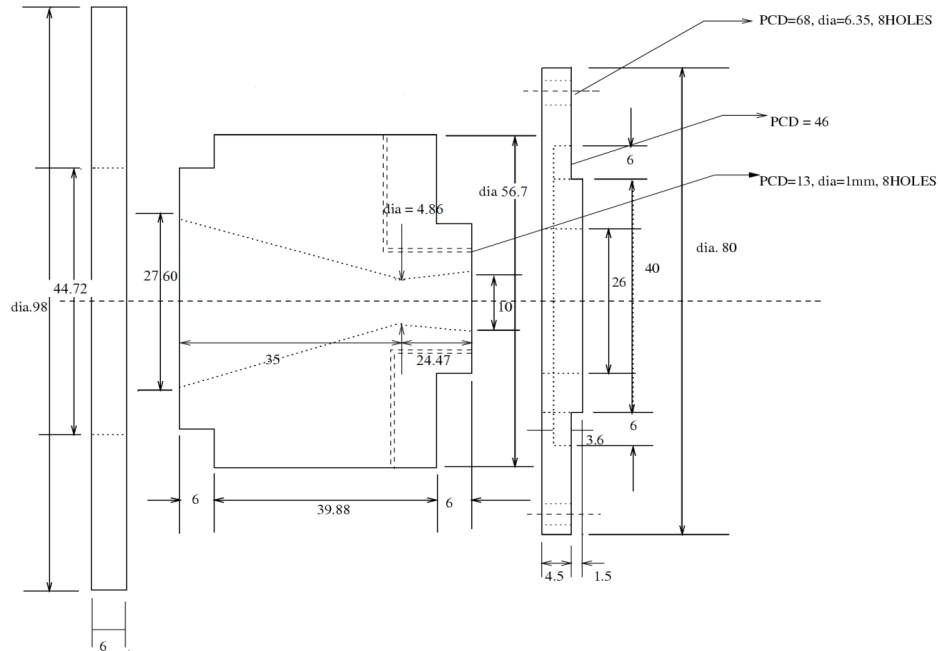


Figure 3 Nozzle designs for machining (Mach 3.0)

Table 1 L_9 orthogonal array

L_9 test	1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

The obtained results are subjected to regression and variance analysis (ANOVA). The regression equation for the current set of experiments can be expressed as

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3, \quad (21)$$

where Y is non-dimensional base pressure, b_0 is the reaction variable of base pressure at the base plane. The coefficients b_1 , b_2 and b_3 are identified with variables viz. Mach number, NPR and L/D respectively which are within selected levels. Taguchi's parameter design provides a systematic and efficient methodology for determining optimum parameters which have an effect on the process and performance. It eliminates the need for repeated experiments and thus saves time, material and cost. In order to observe the influencing degree of process parameters in base pressure measurement, three parameters namely, Mach number M , nozzle

pressure ratio NPR and L/D , each at three levels were considered and are listed in Tab. 2.

Table 2 Factors and their levels

Parameters	Level 1	Level 2	Level 3
Mach number	2.0	2.5	3.0
NPR	5	7	9
L/D	3	5	8

Orthogonal arrays are often employed in industrial experiments to study the effect of numerous control factors. An orthogonal array is a type of experiment wherever the columns for the independent variables are orthogonal to one another. By orthogonal array the analysis becomes simple and leads to large saving in the experiment effort. To describe an orthogonal array, one must identify number of levels and factors. The degrees of freedom (DoF) for three parameters in each of three levels were calculated as number of levels $- 1$. A three-level L_9 orthogonal array with nine experimental runs was selected. The total DoF for the experiment is $9 - 1 = 8$.

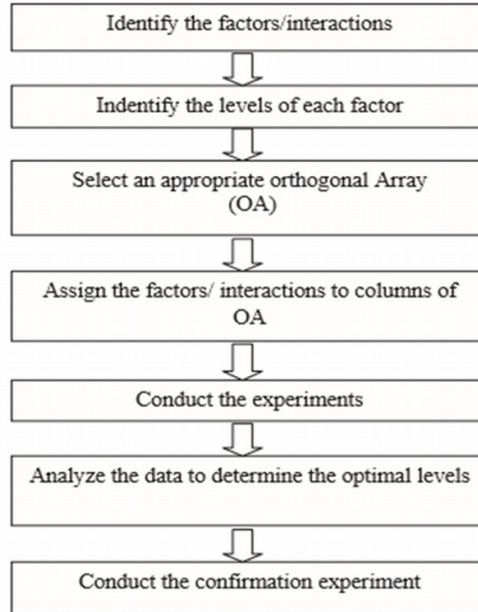


Figure 4 Steps in implementation of Taguchi approach

5. Results and Discussions

The data measured comprises of base pressure P_b distribution for different lengths of the developed channel and the NPR which are characterized by proportion of stagnation pressure P_0 to the back pressure P_{atm} . The measured values of base

pressure were divided by the surrounding air atmospheric pressure in order to be non-dimensionalized. An area ratio of 4.84 has been used in this study and the blow pressure ratio as same as NPR has been used to conduct various runs. Feasibility of using micro-jets in blow mode as a system to control base pressure is the preliminary ambition of the present investigation. This has been done by conducting experiments conventionally in order to study the base pressure variation for Mach numbers of 2.0, 2.5 and 3.0, NPRs of 5, 7 and 9 and L/D s of 3, 5 and 8.

Table 3 Experimental results

S.I. No.	M	NPR	L/D	without control P_b/P_a	with control P_b/P_a
1.	2	5	3	0.59	0.60
2.	2	7	5	0.28	0.29
3.	2	9	8	0.10	0.11
4.	2.5	5	5	0.63	0.62
5.	2.5	7	8	0.37	0.37
6.	2.5	9	3	0.61	0.61
7.	3	5	8	0.72	0.71
8.	3	7	3	0.67	0.66
9.	3	9	5	0.56	0.54

It has been observed from Figs. 5 (a, b and c) that, the nozzle pressure ratio for a given Mach number which controls the level of flow development plays a crucial role on control effectiveness of micro jets. Likewise, it has been seen that as the nozzle pressure ratio increases, the micro jets tend to be more effective in causing considerable variation in the base pressure for the Mach numbers 2.0, 2.5 and 3.0.

From Figs. 5 (b and c), it has been observed that, for Mach numbers 2.5 and 3.0, the control effectiveness for the highest nozzle pressure ratio i.e. NPR=9 has decreased the base pressure all the values of length to diameter ratios. Similarly on the other hand, the base pressure has increased when control is applied for nozzle pressure ratio of 9 for the Mach number 2.0. It must be noted that, the nozzle pressure ratios implemented in the present study are such that the nozzle experiences over, under and correct expansion. It is also known that the expansion fan is generally placed at the nozzle exit for under and over expanded nozzles, respectively. For lower Mach numbers (i.e. Mach= 2.0), this expansion fan at the exit of the nozzle turns the flow away from the base and thereby weakens the base vortex. This phenomenon increases the base pressure as the weak base vortex engages the mass flow infused by micro jets [12,13]. Therefore a further increase in the nozzle pressure ratio (i.e. NPR=9) further weakens the base vortex.

At this juncture when the micro jets are turned on, they inseminate without deflecting, adding mass from the existing vortex and convecting away from the base assuming higher base pressure values when compared to those for without control. However in the case of higher Mach numbers, (i.e. Mach = 2.5, 3.0), although the expansion fan turns the flow away from the base, the turning away tendency of incoming flow is high due to which the base vortex remains stronger. At this point the introduction of micro jets tends to deflect, causing a considerable level of disturbance to the standing vortex which brings about the necessary decrease in base pressure [13]. Therefore for the area ratio (ratio of nozzle exit area to the enlarged duct area) of 4.84 used as per the present study, the use of active control,

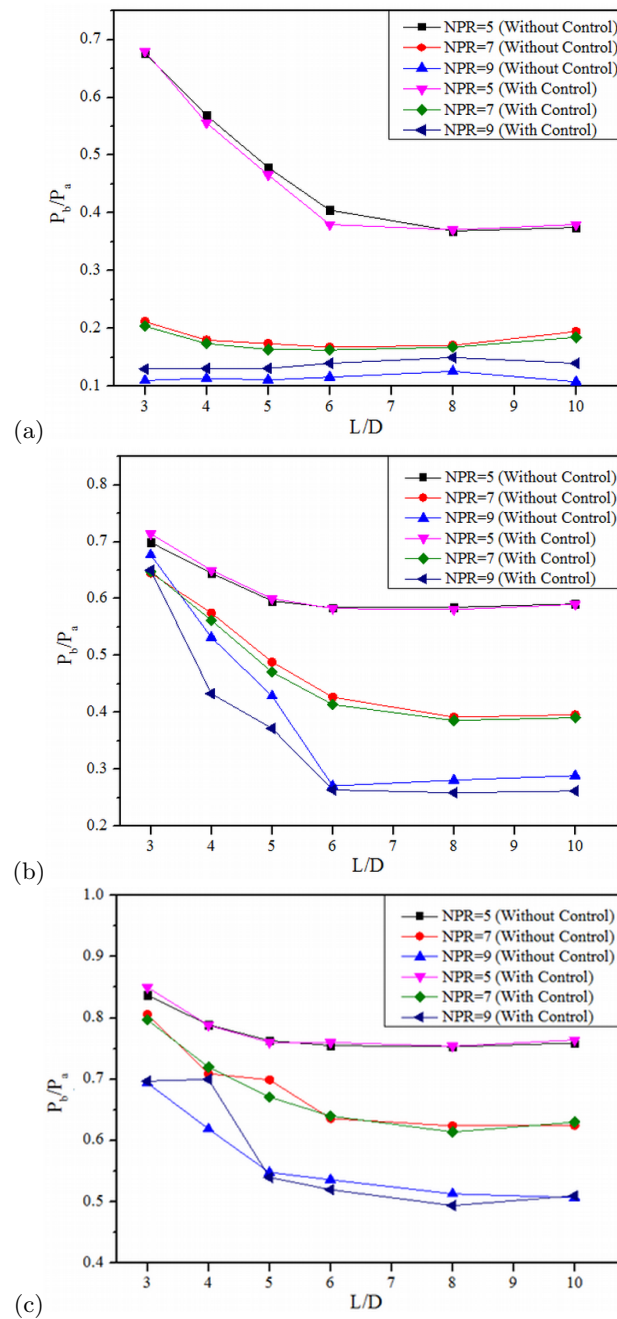


Figure 5 Variation of non-dimensional base pressure with respect to L/D ratio for (a) Mach number 2.0; (b) Mach number 2.5 and (c) Mach number 3.0

1. increases base pressure decreases for higher nozzle pressure ratio of 9 and lower Mach numbers of 2.0;
2. decreases the base pressure for higher Mach numbers of 2.5 and 3.0 and high nozzle pressure ratio of 9.

5.1. Development of Linear Regression Equations

A statistical model on the basis of multiple linear regression equations has been developed for non-dimensional base pressure using the relevant experimental parameters for both cases of without and with control. The standard commercial statistical software MINITAB 17 has been used to derive. The linear polynomial model shown below represents the non-dimensional Base pressure as a function of Mach number, NPR and L/D ratio. The regression equations for both cases of without control and with control are as given below.

For without control

$$P_b/P_a = 0.313 + 0.3267(M) - 0.0558(NPR) - 0.0442(L/D) \quad (22)$$

For with control

$$P_b/P_a = 0.368 + 0.3033(M) - 0.0558(NPR) - 0.0440(L/D) \quad (23)$$

The coefficient of determination R^2 for the case of without control is 90.89% and 90.34% for with control. This is an expected result because the base pressure variations are slightly scattered. The summary of results of the test cases were calculated for the test cases for percentage deviation by using the linear regression equations (22) and (23). It has been found from Tabs. 4 and 5 that, the maximum absolute percentage error is within 25% for without control and 24.13% for with control.

Table 4 Summary of the results of test cases for the response non-dimensional base pressure (without control)

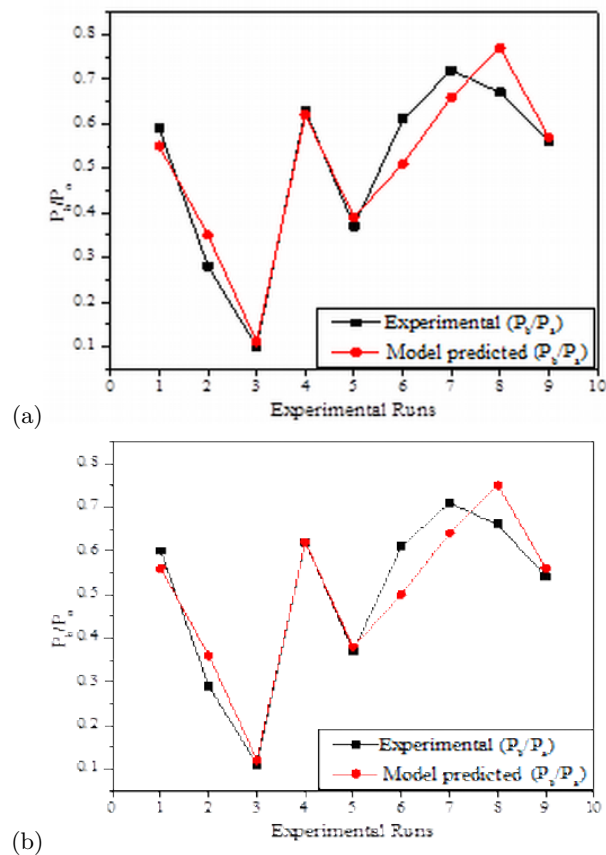
S.I. No.	M	NPR	L/D	Experimental P_b/P_a	Predicted P_b/P_a	Error [%]
1.	2	5	3	0.59	0.55	6.77
2	2	7	5	0.28	0.35	-25
3.	2	9	8	0.10	0.11	-10
4.	2.5	5	5	0.63	0.62	1.58
5.	2.5	7	8	0.37	0.39	-5.40
6.	2.5	9	3	0.61	0.51	16.39
7.	3	5	8	0.72	0.66	8.33
8.	3	7	3	0.67	0.77	-14.92
9.	3	9	5	0.56	0.57	-1.78

The variation of experimental and modal predicted base pressure results for the test cases have been presented in Figs. 6(a) and (b). It is imperative to note that the model predicted non-dimensional base pressure results have agreed well with the experimental results expect for the second experimental test case. The absolute percentage deviation observed for this particular experimental test case was observed to be 25% and 24.13% for without and with control respectively. The

Table 5 Summary of the results of test cases for the response for non-dimensional base pressure (with control)

S.I. No.	M	NPR	L/D	Experiment (P_b/P_a)	Predicted (P_b/P_a)	Err. (%)
1.	2	5	3	0.60	0.56	6.66
2	2	7	5	0.29	0.36	-24.13
3.	2	9	8	0.11	0.12	-9.09
4.	2.5	5	5	0.62	0.62	0
5.	2.5	7	8	0.37	0.38	-2.70
6.	2.5	9	3	0.61	0.50	18.03
7.	3	5	8	0.71	0.64	9.85
8.	3	7	3	0.66	0.75	-13.62
9.	3	9	5	0.54	0.56	-3.70

instrument errors in measurement system and lack of surface finish at the base region produce uneven rate of change base pressure. Also the experimentation is subjected to huge amount of mechanical vibrations as the operations are subjected to high pressure and high velocity conditions.

**Figure 6** Comparison of experimental and modal predicted base pressure results for a) without control and b) with control

This may be the plausible reason for the error, which is experimentally reasonable. Additionally, since the experiments are conducted as per L_9 orthogonal array, the regression equation delivered by the Minitab software is on the basis of these experiments and hence these equations cannot be modified. However in order to ensure the reliability of the developed linear regression equations, confirmation tests have been conducted and have been detailed in the coming section below. The value of b_0 is the intercept of the plane and is the mean reaction value for every experiment conducted. The estimation of b_0 just not relies on leading parameters like M , NPR and L/D which are executed in the study, but also with inconsistencies that occur during experimentation like machine fluctuations, circumstantial conditions and differences in precision machining of both nozzle as well as the enlarged duct. The non-dimensional base pressure calculated from the above said equations consist of positive coefficient values which suggest that base pressure results in increase with increased associated variables, whereas an opposite effect has been observed for negative coefficient values. The magnitudes of such variables indicate relative weight of each factor. The equations clearly suggest that Mach number has a greater effect on base pressure followed by NPR and L/D .

5.2. Confirmation Tests

The linear models developed in the study are tested for their practical significance. For this purpose, test cases were performed randomly and real experiments were conducted to record the base pressure without and with control of the above test cases. The experiments were conducted for selected values for Mach number, NPR, and L/D falling in the respective range of their levels (refer Tab. 2), however were different from those conducted as per L_9 , see Tab. 6. The response wise performance of the linear regression models has been presented. In this view, the line of best fit is used to make the comparison wherein the experimental values are compared with the corresponding modal predicted values (Figs. 7a and b).

Table 6 Input-output data of the confirmation tests

S.I. No.	M	NPR	L/D	Without control P_b/P_a	With control P_b/P_a
1.	2	5	4	0.569	0.556
2.	2	5	6	0.405	0.380
3.	2.5	5	4	0.645	0.650
4.	2.5	5	6	0.584	0.583
5.	3	5	4	0.789	0.788
6.	3	5	6	0.755	0.760
7.	2	7	8	0.443	0.440
8.	2.5	7	6	0.427	0.414
9.	3	7	8	0.806	0.812
10.	2.5	3	3	0.837	0.850

It is observed that the best fit line obtained for base pressure without control shows minimum deviation from the $y = x$ (Fig. 7a) when compared to that for base pressure with control (Fig. 7b). Here the majority of data points lie closer to the ideal line. The estimations of percentage deviation are found to be lying in the range of -6.12% to 10.26% for base pressure without control and -13.92% to 6.58% for base pressure with control (Fig. 8).

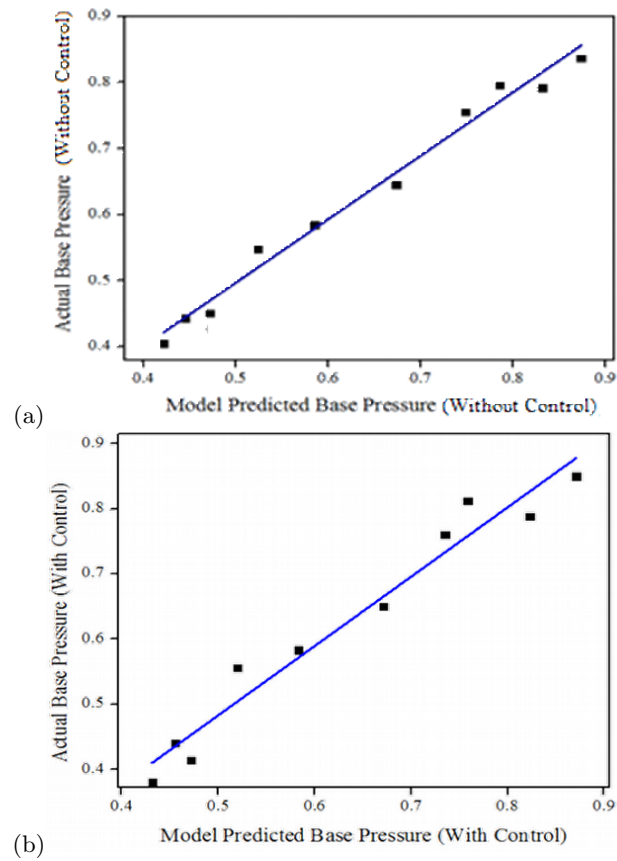


Figure 7 Comparison of model predicted base pressure with actual base pressure: (a) without control; (b) with control

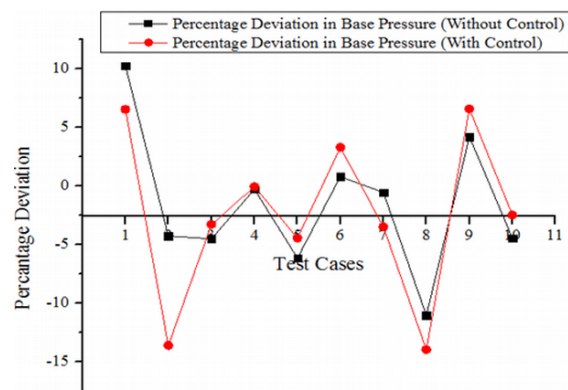


Figure 8 Standard deviation in prediction for 10 cases

It has also been clearly observed that, for the linear regression model of base pressure with control, the data are distributed on either side of the reference line with large amount of variations whereas data points are closely fitted to the reference line in case of the regression model for base pressure without control. It has to be noted that the linear regression model for base pressure without control has demonstrated better prediction with regard to average absolute percentage deviation when compared to base pressure with control and is shown in Fig. 9.

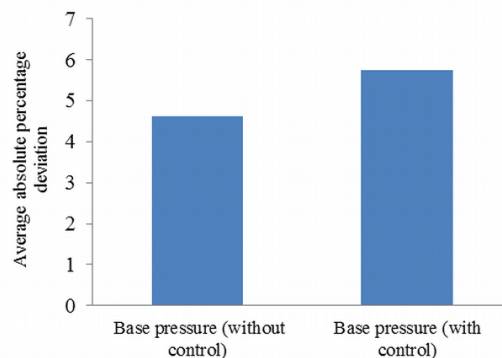


Figure 9 Comparison of linear regression models

5.3. Analysis of Signal to Noise (S/N) Ratio

Non-dimensional base pressure is the response variable which greatly determines the quality of a flow problem. Keeping in view, the interest of its applications, the base pressure should be a minimum in case of combustion chamber in order to maximize mixing and maximum in case of rockets and projectiles to reduce base drag. Thus S/N response for base pressure in the present study has been investigated for the option Smaller the better. This has been largely done in order to facilitate the respective applications. Additionally, the control results in marginal significance over base pressure variation. Hence S/N analysis has been conducted only for the case of without control and with control. From Fig. 10, we find that the optimal parameter for minimum base pressure to be obtained without control is the Mach number at level 1 i.e. (Mach-2), NPR at level 3 (NPR-9) and L/D at level 3 ($L/D = 8$). The S/N analysis for minimum base pressure with control will also constitute the same optimal parameters.

5.4. Analysis of Variance (ANOVA)

The implication of the three factors i.e., M number, NPR and L/D and their interactions by comparing the mean square against an estimate of the non-dimensional base pressure errors at predetermined confidence levels is formally tested with the help of ANOVA. The ANOVA allows analysing the influence of each variable on the total variance of the results. Tables 7 and 8 show the results of ANOVA for the non-dimensional base pressure of the test cases. This analysis was performed with a level of significance of 5% i.e. for a level of confidence of 95%. The last column

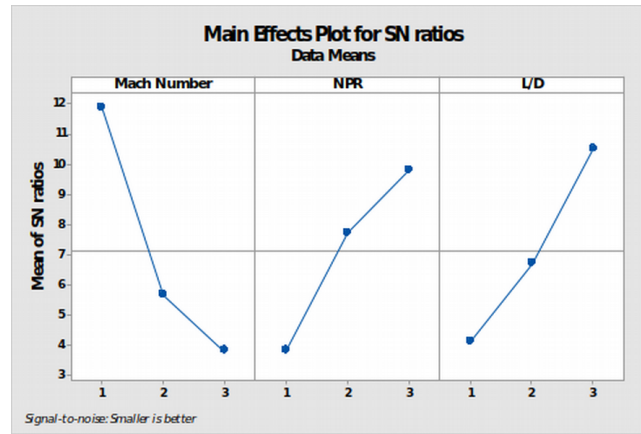


Figure 10 S/N analysis for non-dimensional base pressure without control

of the table shows the contribution $\% (^aP)$ of each variable in the total variation indicating the influence degree on the wear of contact pair. If the F -value is greater than 5%, then the assigned variable is statistically significant. The investigation of factors which affect base pressure significantly is analyzed through analysis of variance.

Source	DF	Adj. SS	Adj. MS	F value	P value	^a P %
M number	2	0.165067	0.082533	22.31	0.043	48.12
NPR	2	0.092867	0.046433	12.55	0.074	27.04
L/D	2	0.077867	0.038933	10.52	0.087	22.68
Error	2	0.007400	0.003700			2.16
Total	8	0.343200				100

Table 7 Analysis of variance for base pressure without control

Source	DF	Adj. SS	Adj. MS	F value	P value	^a P %
M number	2	0.142689	0.071344	17.64	0.054	44.46
NPR	2	0.091622	0.045811	11.33	0.081	28.54
L/D	2	0.078489	0.039244	9.70	0.093	24.42
Error	2	0.008089	0.004044			2.58
Total	8	0.320889				100

Table 8 Analysis of variance for base pressure with control

Table 7 shows that Mach number is having the highest significance of 48.12% on base pressure followed by NPR 27.04% and L/D 22.68 % for the case of without control. Similarly in Table 8, Mach number has highest significance of 44.46% on base pressure followed by NPR 28.54% and L/D 24.42% for the case of with control respectively. Here an important note to be made is that the Mach numbers

employed in the present study belong to the supersonic regime. The NPRs in this regime will usually be over expanded. Higher Mach numbers generally produce a shock at the nozzle exit which tends to turn the flow away from the base. Thus the vortex positioned at the base gets weakened. This vortex in turn brings in major fluctuations in the base pressure at the base which encounters continuous flow of injected mass from the settling chamber at a predetermined pressure [8,10,12]. Thus the influence of Mach number in this particular analysis contributes to a maximum degree when compared to the other factors for both cases.

In the case of NPR, the level of expansion descends with increased NPR thereby, weakening the oblique shock at the exit of the nozzle when compared to low NPRs. In this way, the turning away tendency of approaching flows descends leaving the vortex practically undisturbed. At this stage, if control is presented they may proliferate with no deflecting tendency; consequently the standing vortex avails some mass accordingly convecting it away from the base assuming the base pressure to high values when compared to those for without control. This can be clearly defined as the reason for NPR having more influence in the case of with control 28.54% when compared to the case of without control 27.04%. The pooled error is 2.16% and 2.58% for the cases of without and with control respectively. On the other hand, the influence of L/D (22.68% for without control and 24.42% for with control) is due to the fact that; for L/D s below 5, higher values are assumed for base pressure values for the complete set of Mach numbers that are employed in the present study and has been already is having a length below some limiting value, and is reported earlier by [13]. This is due to the account of the duct when its distance for flow which is suddenly expanded, to reattach and advance downstream is unavailable. This outcomes into non-formation of the strong vortex which otherwise will be situated at the base. This is the reason for high pressure at the base which contributes to its significance.

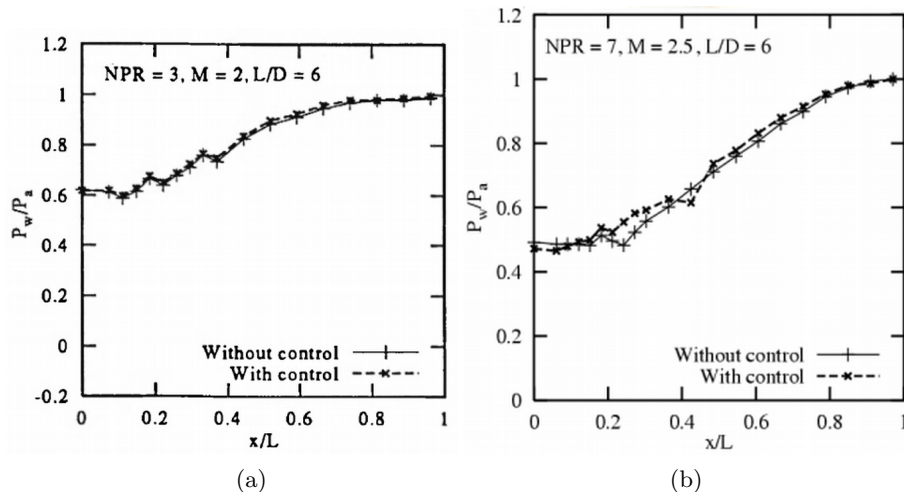


Figure 11 Wall pressure distributions

5.5. Wall Pressure Distribution

It can be seen from Figs. 11 (a and b) that the wall pressure studies are verily required to understand the oscillatory nature of flow which is one of the major problems in active methods of controlling base flows. In other words, it is essential to make sure that the wall pressure field is not adversely influenced (i.e. made oscillatory) by the control for the current set of experiments conducted. To demonstrate this in the present investigation, the wall pressure distribution was measured for various Mach numbers and NPRs. The results of wall pressure distribution as a function jet Mach number and NPR show that the control does not influence the wall pressure.

6. Conclusions

Experimental investigations are carried out to study the control of base pressure without and with the use of micro-jets through suddenly expanded axi-symmetric passage in the supersonic regime as per Taguchi design of experiments. The findings include:

1. Active control becomes more effective for higher NPR s at lower M numbers of 2.0. However decrease in base pressure is observed for higher Mach numbers of 2.5 and 3.0 with active control.
2. Summary of the experimental and model predicted results of the L_9 experimental test cases for the responses non-dimensional base pressure without and with control have been developed by use of the linear regression equations. The maximum percentage error between the experimental and predicted results was found to be $\pm 25\%$.
3. In order to test the developed linear regression equations for their practical significance, confirmation tests were conducted by randomly generated test cases for selected values for Mach number, NPR , and L/D . It has been observed that both the models are reliable and significant with percentage deviation lying in the range of -6.12% to 10.26% for base pressure without control and -13.92% to 6.58% for base pressure with control.
4. The S/N analysis optimal parameter for minimum base pressure to be obtained without control and with control is the Mach number at level 1 i.e. (Mach-2), NPR at level 3 ($NPR - 9$) and L/D at level 3 ($L/D - 8$).
5. Statistical analysis shows that Mach number is found to have the highest significance of 48.12% followed by NPR of 27.04% and L/D of 22.68% for the case of without control. Similarly for the case of with control Mach number is found to have the highest significance of 44.46% followed by NPR of 28.54% and L/D of 24.47% .
6. There is no adverse effect of the active control on the enlarged duct pressure field, as evidenced by the identical behaviour of the wall pressure distribution without and with control.

Nomenclature

T	temperature of ideal gas
A	exit area of the nozzle
U	local flow velocity with respect to the boundaries
A^*	throat area of the
X	pressure measurement across different duct lengths
AR	area ratio
C_p	specific heat of air at constant pressure
γ	ratio of specific heat at constant pressure to constant volume (1.4 for air)
F value	Fisher statistic (Ratio of variances)
ρ	density
M	Mach number at the nozzle exit

Abbreviations

P value	probability of the statistical model
NPR	nozzle pressure ratio
P	static pressure
MS	mean of squares
P_b	base pressure
S/N	signal to noise
P_a	ambient pressure
SS	sum of squares
R	ideal gas constant

Subscripts

a= ambient, e=exit, b= base

References

- [1] **Wick, R.S.:** The effect of boundary layer on sonic flow through an abrupt cross-sectional area change. *Journal of Aeronautical Science*, **20**(10), 675–682, **1953**.
- [2] **Anderson, J.S., Williams, T.J.:** Base pressure and noise produced by the abrupt expansion of air in a cylindrical duct. *Journal of Mechanical Engineering Science*, **10**(3), 262–268, **1968**.
- [3] **Viswanath, P.R., Patil, S.R.:** Effectiveness of Passive Devices for Axisymmetric Base Drag Reduction at Mach 2. *Journal of Spacecraft*, **27**(3), 234–237, **1990**.
- [4] **Badrinarayanan, M.A.:** Experimental investigation of base flows at supersonic speeds. *Journal of Royal Aeronautical Society*, **65**(607), 475–482, **1961**.
- [5] **Khan, S.A., Rathakrishnan, E.:** Active Control of Suddenly Expanded Flows from Over expanded Nozzles. *International Journal of Turbo and Jet Engines*, **19**(1-2), 119–126, **2002**.
- [6] **Khan, S.A., Rathakrishnan, E.:** Control of Suddenly Expanded Flows with Micro Jets. *International Journal of Turbo and Jet Engines*, **20**(1), 63–81, **2003**.
- [7] **Khan, S.A., Rathakrishnan, E.:** Active Control of Suddenly Expanded Flow from Under Expanded Nozzles. *International Journal of Turbo and Jet Engines*, **21**(4), 233–253, **2004**.

- [8] **Khan, S.A., Rathakrishnan, E.:** Control of Suddenly Expanded Flow from Correctly Expanded Nozzles. *International Journal of Turbo and Jet Engines*, **21**(4), 255–278, **2004**.
- [9] **Khan, S.A., Rathakrishnan, E.:** Control of Suddenly Expanded Flow. *Aircraft Engineering and Aerospace Technology: International Journal*, **78**(4), 293–309, **2006**.
- [10] **Baig, M.A.A., Al-Mufadi, F., Khan, S.A., Rathakrishnan, E.:** Control of Base Flows with Micro jets. *International Journal of Turbo and Jet Engines*, **28**(1), 59–69, **2011**.
- [11] **Cantwell, B.J.:** *Fundamentals of Compressible Flow. AA210*, Department of Aeronautics and Astronautics, Stanford University, California, USA, **1996**.
- [12] **Rehman S., Khan, S.A.:** Control of base pressure with micro-jets: part I. *Aircraft Engineering and Aerospace Technology: International Journal*, **80**(2), 158–164, **2008**.
- [13] **Rathakrishnan, E., Sreekanth, A.K.:** Flow in pipes with sudden enlargement, in: *Proceedings of the 14th International Symposium Space Technology and Science*, Tokyo, Japan, 491–496, **1984**.

